3. Optimization Problem of UEP scheme for Uncompressed Video Transmission

\begin{figure}[htb]

\centering

\includegraphics[width=\textwidth]{fig/video\_communication\_structure.eps}

\caption{\label{fig:vc\_structure}A conventional structure of a wireless uncompressed video communication system.}

\end{figure}

In Fig. ~\ref{fig:vc\_structure}, a high-level conceptual structure of a wireless uncompressed video communication system is shown. In a conventional uncompressed video communication system, the uncompressed input sequence is usually split into frames, and each video frame is usually divided into $m$ bit-planes since all pixels in this frame can be represented by $m$ bits (e.g. 8 bits representation for each color channel is the most popular format). The bit stream composed of each bit-plane is encoded by the channel encoder to increase information redundancy, in order to reduce the error from the wireless error-prone channel. The encoder with Equal error protection (EEP) scheme would allocate the same amount of transmission power for each bitstream from any level of bit-plane. The transmission power for a signal directly affects the average bit error rate (BER) of the received and channel-decoded but distorted signal. However, the importance of each bit-plane for the visual quality of a frame is unequal. For the purpose of higher decoded video quality, better transmission power distribution for each bit-plane should be derived.

As a result, in this chapter, the basic unequal error protection (UEP) scheme designed for uncompressed video is presented in Section ~\ref{cc:bitplane}. After the optimization problem is formulated, the solving method with the aid of largrange multiplier is derived in Section ~\ref{cc:contrained\_opt}. Finally, the numerical result is discussed in the Subsection ~\ref{ccc:res}.

%==========================================================%

3.1 UEP Scheme at Bit-Plane Level of Uncompressed Video

\begin{figure}[htb]

\centering

\includegraphics[width=\textwidth]{fig/sm.pdf}

\caption{\label{fig:uep\_encoder}A structure of the wireless uncompressed video communication system with UEP scheme.}

\end{figure}

The encoder with the proposed UEP scheme is depicted in Fig. ~\ref{fig:uep\_encoder}. At the time instant $\tau$, a video frame $F\_k$ with incremental index $k$ is read from the video source. The set consisting of the pixels from $F\_k$ is denoted as

\begin{equation}

\mathbf{f}\_k = \{f\_k^{(x,y)}|x\in\{1,\cdots,w\},y\in\{1,\cdots,h\}\},

\end{equation}

where $(w, h)$ are the width and height of the video frame $F\_k$. Each element of $\mathbf{f}\_k$ is an $m$-bit unsigned integer, whose value ranges from $0$ to $2^{m}-1$. Since each pixel value can be represented in $m$ binary bits, a video frame can be divided into $m$ bit-planes in the order of its signification, and the $n$-th bit-plane can be denoted as

\begin{equation}

\mathbf{u}\_{k,n}=\{u\_{k.n}^{(x,y)}=f\_k^{(x,y)}(n)|\forall f\_k^{(x,y)}\in \mathbf{f}\_k\},\qquad n\in\{1,\cdots,m\},

\end{equation}

where $ f\_k^{(x,y)}(n)$ is refer to the $n$-th bit of $f\_k^{(x,y)}$. It is worth noting that $n=1$ is the least significant bit (LSB) level having the least impact to the pixel value while $n=m$ is the most significant bit (MSB) level with the greatest influence. Then, the bit-planes are arranged in the order from LSB level to MSB level.

An example of generation of bit-planes from a frame is shown in Fig. ~\ref{fig:bitplane}.

\begin{figure}[htb]

\centering

\includegraphics[scale=0.3]{fig/bitplane.pdf}

\caption{\label{fig:bitplane}A video frame can be divided into bit-planes in the order of its signification.}

\end{figure}

Each bit-plane is then scanned into serial bit sequence and then the channel code is applied to the bit sequence of one bit-plane. In EEP scheme in Fig.~\ref{fig:vc\_structure}, the transmission power allocated for each channel-coded bit

\begin{equation}

\mathbf{c}\_{k,n}=[c\_{k,n}^1, c\_{k,n}^2,\cdots, c\_{k,n}^{h\cdot w/R\_c}] \nonumber

\end{equation}

is same and denoted by $E\_b$, where $R\_c$ is the code rate of the applied channel code, that is, the transmission power for any bit in $\{\mathbf{u}\_{k,n}|n\in\{1,\cdots,m\}\}$ is same as

\begin{align}

E\_s=\frac{E\_b}{R\_c}. \nonumber

\end{align}

However, the proposed UEP scheme is to distribute different transmission power to bit-planes $\{\mathbf{u}\_{k,n}|n\in\{1,\cdots,m\}\}$ according to their prioritization and the redundancy, in order to improve the visual quality of the received sequence. The evaluation metric for visual quality is peak signal-to-noise ratio (PSNR) represented as

\begin{equation}

PSNR=20log\_{10}(\frac{2^m-1}{\sqrt{D}}),

\label{eq:psnr}

\end{equation}

where $D$ is the average pixel distortion in terms of mean squared error, that is

\begin{equation}

D=\frac{1}{h\cdot w}\sum\_{i=0}^{h\cdot w}{(f\_k^i-\hat{f}\_k^i)^2},

\label{eq:mse}

\end{equation}

where $f\_k^i$ is the $i$-th pixel value in $\mathbf{f}\_k$, and $\hat{f}\_k^i$ is the corresponding decoded pixel value at receiver. As a result, from( ~\ref{eq:psnr}), higher quality requires lower pixel distortion $D$ in (~\ref{eq:mse}). Besides, the mismatch between pixel value and decoded pixel value is directly related to the channel BER, which depends on the applied channel code, modulation and the signal-to-noise ratio (SNR) $\gamma\_0=E\_b/N\_0$.

Since each bit-level has different impact to its final pixel value $f\_k^i$, UEP scheme should determine the transmission power for each bit-plane as depicted in Fig.~\ref{fig:uep\_encoder}

\begin{align}

E\_{bn} = & R\_s\cdot E\_{sn} \nonumber \\

= & R\_s\cdot{(w\_n\cdot E\_{s0})} \nonumber \\

= & w\_n\cdot E\_{b0},

\label{eq:weighted\_energy}

\end{align}

where $E\_{bn}$ is the transmission power for every channel-coded bits, $E\_{s0},\quad E\_{b0}$ is a target or assigned average transmission power, and $w\_n$ is the ratio of the target power to the determined transmission power for $n$-th bit-plane. UEP scheme takes the frame information and the channel information and decides the weights $\{w\_n|n\in\{1,\cdots,m\}\}$ for each bit-plane, in order to generate the optimized expected visual quality, that is the least expected average distortion,

\begin{equation}

\mathbf{w}^{\ast}\equiv [w\_1^{\ast},w\_2^{\ast},\cdots,w\_m^{\ast}]^T = \arg\min\_{\mathbf{w}}{E[\hat{D}(\gamma\_0,\mathbf{w},\mathbf{f}\_k)]},

\label{eq:uep\_general\_eq}

\end{equation}

where $\mathbf{w}$ is the vector of weights for every bit-level, $E[\cdot]$ denotes the expectation function, and $\hat{D}(\gamma\_0,\mathbf{w},\mathbf{f}\_k)$ is the estimated average distortion with the input parameters, transmission power for each bit-planes, channel noise, and frame information.

3.1.1 Estimation of Decoded Video Quality

According to (\ref{eq:bitlevel}), the distortion equation in (\ref{eq:mse}) can turn into

\begin{align}

D = &\frac{1}{h\cdot w}\sum\_{i=0}^{h\cdot w}{(\sum\_{n=1}^{m}{2^{n-1}\cdot u\_{k, n}^i}-\sum\_{n=1}^{m}{2^{n-1}\cdot \hat{u}\_{k, n}^i})^2},\nonumber \\

= &\frac{1}{h\cdot w}\sum\_{i=0}^{h\cdot w}{\{ \sum\_{n=1}^{m}{\sum\_{l=1}^{m}{[2^{n+l-2}\cdot(u\_{k,n}^i u\_{k,l}^i-2u\_{k,n}^i\hat{u}\_{k,l}^i+\hat{u}\_{k,n}^i\hat{u}\_{k,l}^i)]}}\}},

\label{eq:distortion\_bitlevel}

\end{align}

where $\hat{f}\_k^i=\sum\_{n=1}^{m}{2^{n-1}\cdot \hat{u}\_{k, n}^i}$ represents that $\hat{u}\_{k,n}^i$ is the $n$-th bit of decoded pixel $\hat{f}\_k^i$. However, the received and channel-decoded bit-planes $\hat{\textbf{u}}\_{k,n}$ are random variables due to the error-prone channel. As a result, a function refer to the channel bit error rate is applied

\begin{equation}

fr(E\_b, N\_0, \mathit{channel\ \ code}, \mathit{modulation}) = P(\hat{u}\_{k,n}^i \neq u\_{k,n}^i |u\_{k,n}^i),

\label{eq:ber\_def}

\end{equation}

where {\it channel code} and {\it modulation} respectively denote the applied channel code and the applied modulation method. Since proposed UEP scheme only adjusts the $E\_b$ to increase the performance, (\ref{eq:ber\_def}) is abbreviated to $fr(\gamma\_b)$, where $\gamma\_b=E\_b/N\_0$ is refer to the SNR.

Since the BER function in (\ref{eq:ber\_def}) can be simulated in advance in the condition of the fixed channel code and modulation method, random variables $\hat{\textbf{u}}\_{k,n}$ can be predicted in the encoder with probability

\begin{equation}

P(\hat{u}\_{k,n}^i = u) = \left\{\begin{array}{ll}

fr(\gamma\_{bn} = \frac{w\_n\cdot E\_{b0}}{N\_0}) & ,\quad u \neq u\_{k,n}^i \\

1- fr(\gamma\_{bn} = \frac{w\_n\cdot E\_{b0}}{N\_0}) & ,\quad u = u\_{k,n}^i \\

0 & ,\quad otherwise

\end{array}\right.

\label{eq:est\_u}

\end{equation}

Furthermore, the expectation of the average pixel distortion in (\ref{eq:distortion\_bitlevel}) is denoted by

\begin{equation}

E[D] = \frac{1}{4N}\sum\_{i=1}^{N} {\{ \sum\_{n=1}^{m}{\sum\_{l=1}^{m}{(2^{n+l}\cdot E[u\_{k,n}^i u\_{k,l}^i-2u\_{k,n}^i\hat{u}\_{k,l}^i+\hat{u}\_{k,n}^i\hat{u}\_{k,l}^i])}}\}},

\label{eq:distortion\_exp}

\end{equation}

where $N=h\cdot w$ is the total number of pixels in a frame. $E[\cdot]$ terms in above equation (\ref{eq:distortion\_exp}) can be first partitioned into two parts, where one part is in the condition of $n=l$ and the other is about $n\neq l$:\\

\begin{enumerate}[topsep=0pt, itemsep=0pt, label=\arabic{\*}.]

\item \texttt{$n=l$:}\\ In this part,

\begin{align}

E[\cdot] = & E[u\_{k,n}^i(u\_{k,n}^i- \hat{u}\_{k,n}^i)] + E[\hat{u}\_{k,n}^i(\hat{u}\_{k,n}^i- u\_{k,n}^i)] \nonumber \\

= & P(u\_{k,n}^i=1)\cdot fr(\gamma\_{bn}) + P(\hat{u}\_{k,n}^i=1)\cdot fr(\gamma\_{bn}),

\label{eq:distortion\_n\_eq\_l}

\end{align}

where $P(\hat{u}\_{k,n}^i=1)$ can be modified as

\begin{align}

P(\hat{u}\_{k,n}^i=1)= & E[\hat{u}\_{k,n}^i] \nonumber \\

= & fr(\gamma\_{bn})\cdot P(u\_{k,n}^i=0) + (1-fr(\gamma\_{bn})\cdot P(u\_{k,n}^i=1) \nonumber \\

= & (P(u\_{k,n}^i=1)+(1-2 P(u\_{k,n}^i=1))\cdot fr(\gamma\_{bn})).

\label{eq:expected\_hat\_u}

\end{align}

By combining (\ref{eq:distortion\_n\_eq\_l}) and (\ref{eq:expected\_hat\_u}), (\ref{eq:distortion\_n\_eq\_l}) becomes

\begin{equation}

E[\cdot] = fr(\gamma\_{bn})\cdot(2P(u\_{k,n}^i=1)+(1-2P(u\_{k,n}^i=1))\cdot fr(\gamma\_{bn})).

\label{eq:distortion\_n\_eq\_l\_1}

\end{equation}

Then from the fact that $P(u\_{k,n}^i=1)= P(u\_{k,n}^i=0)=0.5$, (\ref{eq:distortion\_n\_eq\_l\_1}) can further be derived as

\begin{align}

E[\cdot] = fr(\gamma\_{bn}).

\label{eq:distortion\_n\_eq\_l\_2}

\end{align}

\item \texttt{$n\neq l$:}\\ On the other hand, $n\neq l$ implies that the correlation between two difference bit levels in the same pixel value should be investigated. For simplicity, the correlation can be seen as uncorrelated since the bits in difference levels are almost independent. Thus, $E[\cdot]$ in this part becomes

\begin{align}

E[\cdot] = & E[u\_{k,n}^i]E[u\_{k,l}^i]-2E[u\_{k,n}^i]E[\hat{u}\_{k,l}^i]+E[\hat{u}\_{k,n}^i]E[\hat{u}\_{k,l}^i],

\label{eq:distortion\_n\_neq\_l}

\end{align}

According to $P(u\_{k,n}^i=1)= P(u\_{k,n}^i=0)=0.5$ and (\ref{eq:expected\_hat\_u}), (\ref{eq:distortion\_n\_neq\_l}) can be calculated

\begin{equation}

E[\cdot] = 0.5\cdot 0.5 – 2\cdot 0.5\cdot 0.5 + 0.5\cdot 0.5 = 0.

\label{eq:distortion\_n\_neq\_l\_2}

\end{equation}

\end{enumerate}

Finally, substituting (\ref{eq:distortion\_n\_eq\_l\_2}) and (\ref{eq:distortion\_n\_neq\_l\_2}) for $E[\cdot]$ term in (\ref{eq:distortion\_exp}) and obtaining the expected average pixel distortion as

\begin{align}

E[\hat{D}(\gamma\_0,\mathbf{w})] = & \frac{1}{4N}\sum\_{i=1}^{N} { \sum\_{n=1}^{m}{(2^{2n}\cdot fr(\gamma\_{bn}))}} \nonumber \\

= & \frac{1}{4}\sum\_{n=1}^{m}{(2^{2n}fr(\gamma\_{bn}))}.

\label{eq:distortion\_final}

\end{align}

3.2 Solving Constrained Optimization Problem with Lagrange Multiplier

\label{cc:contrained\_opt}

3.2.1 Minimizing the Video Distortion with Given Transmission Power Constraint

\label{ccc:problem-maxpsnr}

Assuming that the average transmission power for each bit is given, proposed UEP scheme is expected to reach the goal to control the weights $\{\textbf{w}\_n|n\in\{1,\cdots,m\}\}$ of transmission power in different bit-levels in order to maximize the received video visual quality in (\ref{eq:psnr}). This optimization problem can be formally described as

\begin{equation}

\begin{aligned}

& \underset{\mathbf{w}}{\text{minimize}} & & {E[\hat{D}(\gamma\_0,\mathbf{w})]} \\

& \text{subject to} & & \sum\_{n=1}^{m}{w\_nE\_0} = mE\_0, \\

& \text{that is} &\rightarrow & \sum\_{n=1}^{m}{w\_n} = m,

\end{aligned}

\label{eq:opt\_problem\_d}

\end{equation}

where $E\_0$ is the target average transmission power for a bit before channel coding, $\gamma\_0$ is the target average SNR,

\begin{align}

\gamma\_0=\frac{R\_s\cdot E\_0}{N\_0}, \nonumber

\end{align}

and $\textbf{w}^{\ast}$ are the weights able to minimize the expected video distortion in (\ref{eq:distortion\_final}).

To solve this optimization problem and to meet the average transmission power constraint, a lagrange multiplier $\lambda$ is introduced and (\ref{eq:distortion\_final}) is combined into (\ref{eq:opt\_problem\_d}) as

\begin{equation}

\underset{\{ \mathbf{w},\lambda \}}{\text{minimize}} \quad J\_{minD} = \frac{1}{4}\sum\_{n=1}^{m}{(2^{2n}fr(w\_n\gamma\_0))} + \lambda(\sum\_{n=1}^{m}{w\_n}-m),

\label{eq:opt\_lagrange\_minD}

\end{equation}

where $J\_{minD}$ is the lagrangian cost function to be minimized with arguments $(\textbf{w},\lambda)$, and $fr(\gamma)$ is an known BER function which is simulated in advance.

Supposing $\textbf{w}^{\ast}$ is a set of optimized solution to meet (\ref{eq:opt\_problem\_d}), then there exists $\lambda^\ast$ such that $(\textbf{w}^{\ast},\lambda^\ast)$ is a critical point of $J\_{minD}$. In other words, the gradient values of $J\_{minD}$ at $(\textbf{w}^{\ast},\lambda^\ast)$ are all 0, that is

\begin{equation}

\nabla\_{(\mathbf{w},\lambda)}J\_{minD}|\_ {\substack{\mathbf{w} = \mathbf{w}^{\ast} \\ \lambda = \lambda^\ast}}=\mathbf{0}.

\label{eq:lagrange\_gradient\_minD}

\end{equation}

Hence, the system of the partial derivative equations is yielded as

\begin{equation}

\left \{ \begin{array}{ll}

\begin{aligned}

\frac{\partial J\_{minD}}{\partial \lambda} & = & &(\sum\_{n=1}^{m}w\_n)-m & = & &0 & \\

\frac{\partial J\_{minD}}{\partial w\_n} & = & & 2^{2n-2}\frac{\partial fr(w\_n\gamma\_0)}{\partial w\_n} + \lambda & = & &0 &, \quad \forall n \in \{1,\cdots,m\},

\end{aligned}

\end{array}\right.

\label{eq:lagrange\_derivative\_minD}

\end{equation}

where the first equation is just the constraint on the average transmission power, and the second equation can be rearranged as

\begin{equation}

\frac{\partial fr(w\_n\gamma\_0)}{\partial (w\_n\gamma\_0)}=-\frac{\lambda}{\gamma\_0 2^{2n-2}}, \forall n \in \{1,\cdots,m\}.

\label{eq:lagrange\_second\_minD}

\end{equation}

Noting that the BER versus SNR function $fr(\gamma)$ is known, and this curve is always can be upper bounded by some exponential function, which is second-order differentiable and monotonic decreasing. Hence, (\ref{eq:lagrange\_second\_minD}) can be further modified to represent $w\_n$ as a function of parameter $\lambda$

\begin{equation}

w\_n = \frac{Df^{-1}(-\frac{\lambda}{\gamma\_0 2^{2n-2}})}{\gamma\_0}, \forall n \in \{1,\cdots,m\},

\label{eq:lagrange\_w\_n}

\end{equation}

where $Df(\gamma)=\frac{\partial fr(\gamma)}{\partial \gamma}$ is the derivative of $fr(\gamma)$, and $Df^{-1}(\cdot)$ denotes the inverse function of $Df(\gamma)$. Then, substituting (\ref{eq:lagrange\_w\_n}) into the constraint equation (\ref{eq:lagrange\_derivative\_minD})

\begin{equation}

\sum\_{n=1}^{m}{Df^{-1}(-\frac{\lambda}{\gamma\_0 2^{2n-2}})}=m\gamma\_0,

\label{eq:lagrange\_constraint}

\end{equation}

where the value of $\lambda$ can be solved trivially if the close form of $f(\gamma)$ is exist. However, $f(\gamma)$ is usually acquired through the simulation in advance. As a result, $\lambda$ is usually solved by newton's method or other root-finding algorithms because there is not a close-form expression for the simulated and fitted curve $fr(\gamma)$.

\begin{equation}

\text{set } F(\lambda) \equiv (\sum\_{n=1}^{m}Df^{-1}(-\frac{\lambda}{\gamma\_0 2^{2n-2}}))-m\gamma\_0,

\label{eq:newton\_method\_function}

\end{equation}

and newton's method updates the value of $\lambda$ in each iteration according to the slope at current $\lambda$ as below

\begin{equation}

\lambda\_{n+1} \leftarrow \lambda\_{n} - \alpha\frac{\Delta\lambda \cdot F(\lambda\_{n})}{F(\lambda\_{n}+\Delta\lambda)-F(\lambda\_{n})},

\label{eq:newton\_method\_lambda}

\end{equation}

where $\alpha$ is a convergent rate for the newton's method, and $\Delta\lambda$ is a small amount of shift on $\lambda$ to approximate the first derivative value. (\ref{eq:newton\_method\_lambda}) makes $\lambda\_{n}$ gradually converge to $\lambda^{\ast}$. After solving the $\lambda^{\ast}$ in (\ref{eq:lagrange\_constraint}), all weights $\{w\_n^{\ast}|n\in\{1,\cdots,m\}\}$ are then able to be solved through substituting $\lambda^{\ast}$ into (\ref{eq:lagrange\_second\_minD}).

3.2.2 Minimizing the Required Power with Certain Video Distortion Constraint

\label{ccc:problem-minpower}

Similar to the procedure in the previous section \ref{ccc:problem-maxpsnr}, firstly formulating the optimization problem as following

\begin{equation}

\begin{aligned}

& \underset{\{\mathbf{w},E\_{0}\}}{\text{minimize}} & & \sum\_{n=1}^{m}{w\_nE\_{0}} \\

& \text{subject to} & & E[\hat{D}(\frac{R\_sE\_{0}}{N\_0},\mathbf{w})] = D\_0,

\end{aligned}

\label{eq:lagrange\_problem\_minE}

\end{equation}

where $\sum\_{n=1}^{m}{w\_nE\_{0}}$ is the total transmission power expected to be minimized, and the constraint is the estimated expected visual distortion is equal to given quality $D\_0$. Furthermore, $(\textbf{w},E\_{0})$ can be reduced into $\textbf{E}=\{E\_n=w\_nE\_0|n\in\{1,\cdots,m\}\}$, and (\ref{eq:lagrange\_problem\_minE}) is modified as

\begin{equation}

\begin{aligned}

& \underset{\textbf{E}}{\text{minimize}} & & \sum\_{n=1}^{m}{ E\_{n}} \\

& \text{subject to} & & \frac{1}{4}\sum\_{n=1}^{m}{(2^{2n}fr(\frac{R\_sE\_{n}}{N\_0}))} = D\_0.

\end{aligned}

\label{eq:lagrange\_problem2\_minE}

\end{equation}

To solve this constrained optimization problem, lagrange multiplier method is adopted again as in previous problem,

\begin{equation}

\underset{\{ \textbf{E},\lambda \}}{\text{minimize}} \quad J\_{minE} = \sum\_{n=1}^{m}{ E\_{n}} + \lambda(\frac{1}{4}\sum\_{n=1}^{m}{(2^{2n}fr(\frac{R\_sE\_{n}}{N\_0}))}-D\_0),

\label{eq:opt\_lagrange\_minE}

\end{equation}

where $J\_{minE}$ is the lagrangian cost function to be minimized, $\lambda$ is the introduced lagrange multiplier parameter to meet the constraint on the expected visual distortion.

Supposing $(\textbf{E}^{\ast},\lambda^{\ast})$ is the solution of (\ref{eq:opt\_lagrange\_minE}), then the gradient values of $J\_{minE}$ at $(\textbf{E}^{\ast},\lambda^{\ast})$ are all 0 as shown below

\begin{equation}

\nabla\_{(\textbf{E},\lambda)}J\_{minE}|\_ {\substack{\textbf{E} = \textbf{E}^{\ast} \\ \lambda = \lambda^\ast}}=\mathbf{0}.

\label{eq:lagrange\_gradient\_minE}

\end{equation}

Hence, (\ref{eq:lagrange\_gradient\_minE}) implies the system of the partial derivative equations,

\begin{equation}

\left \{ \begin{array}{ll}

\begin{aligned}

\frac{\partial J\_{minE}}{\partial \lambda} & = & &\frac{1}{4}\sum\_{n=1}^{m}{(2^{2n}fr(\frac{R\_sE\_{n}}{N\_0}))}-D\_0 & = & &0 & \\

\frac{\partial J\_{minE}}{\partial E\_n} & = & & 1+\frac{\lambda 2^{2n-2}}{N\_0}\frac{\partial fr(E\_n/N\_0)}{\partial (E\_n/N\_0)} & = & &0 &, \quad \forall n \in \{1,\cdots,m\},

\end{aligned}

\end{array}\right.

\label{eq:lagrange\_derivative\_minE}

\end{equation}

where the first equation is just the constraint on expected distortion, and the second equation can be rearranged similar to (\ref{eq:lagrange\_second\_minD}) and (\ref{eq:lagrange\_w\_n}) and become

\begin{equation}

E\_n=N\_0\cdot Df^{-1}(\frac{-N\_0}{\lambda 2^{2n-2}}),\quad \forall n \in \{1,\cdots,m\}.

\label{eq:lagrange\_E\_n}

\end{equation}

where $Df^{-1}(\cdot)$ is same with that in (\ref{eq:lagrange\_w\_n}). After substituting (\ref{eq:lagrange\_E\_n}) into the constraint equation (\ref{eq:lagrange\_derivative\_minE}) to obtain

\begin{equation}

\sum\_{n=1}^{m}{2^{2n-2}fr(Df^{-1}(\frac{-N\_0}{\lambda 2^{2n-2}})}=D\_0.

\label{eq:lagrange\_constraint\_minE}

\end{equation}

Similar to dealing with (\ref{eq:lagrange\_constraint}), newton’s method is again adopted to solve the $\lambda^{\ast}$.

\begin{equation}

\text{set } F(\lambda) \equiv (\sum\_{n=1}^{m}{2^{2n-2}fr(Df^{-1}(\frac{-N\_0}{\lambda 2^{2n-2}})})-D\_0,

\label{eq:newton\_method\_function\_minE}

\end{equation}

where newton's method updates the value of $\lambda$ in each iteration according to the slope at current $\lambda$ as same as in (\ref{eq:newton\_method\_lambda}). After the value of $\lambda$ converges to $\lambda^{\ast}$, the final value of $\lambda$ is substituted into (\ref{eq:lagrange\_E\_n}) to solve all the elements in $\{E\_n^{\ast}|n\in\{1,\cdots,m\}\}$.

3.2.3 Numerical Result

\label{ccc:res}

% Environment setting

We evaluate the both proposed UEP schemes by comparing with the performance of EEP scheme. A wireless uncompressed video transmission system in Fig. \ref{fig:uep\_encoder} is utilized to be the simulation scenario. Energy distribution for different bit-planes is executed both in EEP and UEP schemes before channel code encoder. Nevertheless, the EEP scheme delivers the same power for each bit-plane while the UEP scheme allocates power according to the methods derived in sections \ref{ccc:problem-maxpsnr} and \ref{ccc:problem-minpower}. The recursive systematic convolutional code with the generator matrix

\begin{equation}

\mathbf{G}(D)=[1 \ \ \frac{1+D^2}{1+D+D^2}]

\label{eq:generator\_matrix}

\end{equation}

is adopted as the channel correction code. The wireless signals are modulated by BPSK, and the channel noise is assumed to be additive white Gaussian noise (AWGN). Besides, the receiver only directly decodes the channel code with the BCJR algorithm \cite{BCJR} introduced in section \ref{bbb:BCJR}.

First of all, the estimation for the decoded video distortion derived in section \ref{cc:est\_quality} is verified through comparing the estimated distortion with simulated average distortion in different SNR conditions with EEP scheme. The evaluation metric is PSNR referred in (\ref{eq:psnr}), and the video clip in raw format contains 1920x1080 pixels in a frame and only 8-bit ($m=8$) luminance color channel is considered for a pixel. In each average SNR conditions, there are 300 frames to be transmitted to obtain the average simulated PSNR. Besides, in EEP scheme, distortion estimation in (\ref{eq:distortion\_final}) is simplified as

\begin{equation}

\begin{aligned}

E[\hat{D}(\gamma\_0)] = &\frac{1}{4}\sum\_{n=1}^{m}{(2^{2n}fr(\gamma\_0))}, \nonumber \\

= & 21845\cdot fr(\gamma\_0)

\end{aligned}

\label{eq:distortion\_est\_eep}

\end{equation}

where $\gamma\_0$ is the given average ratio of transmission power to noise power, expected distortion is only depend on $\gamma\_0$, and BER empirical curve $fr(\gamma)$ is depicted in Fig. \ref{fig:channel\_fr}.

\begin{figure}[!htb]

\centering

\includegraphics[width=0.8\textwidth]{fig/channel\_fr.pdf}

\caption{\label{fig:channel\_fr}A simulated BER versus SNR curve for convolution code in (\ref{eq:generator\_matrix}) and BCJR decoding.}

\end{figure}

% Distortion Estimation / Simulation

\begin{figure}[thb]

\centering

\includegraphics[width=\textwidth]{fig/psnr\_sim\_est.pdf }

\caption{\label{fig:verification\_distortion\_est} Simulated average PSNR and Estimated PSNR versus different SNRs.}

\end{figure}

From Fig.~\ref{fig:verification\_distortion\_est}, the validity of the derived estimation for visual distortion is significant. The root-mean-square deviation of estimated distortion with respect to the simulated distortion is only $0.286$ dB, and thus (\ref{eq:distortion\_final}) can be a proper distortion estimator in the encoder.

% Simulation EEP / UEP\_D

After verifying that the proposed visual distortion estimator is precise enough, the UEP scheme for the optimized video quality under the constraint of the fixed average transmission energy can be built according to (\ref{eq:lagrange\_w\_n})-(\ref

{eq:newton\_method\_lambda}). Besides, the result of EEP scheme is same as the simulated curve in Fig. ~\ref{fig:verification\_distortion\_est}. Therefore, we can then compare the performance gain of the UEP scheme in different SNRs conditions.

\begin{figure}[!htb]

\centering

\includegraphics[width=\textwidth]{fig/ psnr\_direct\_uep\_eep\_d.pdf }

\caption{\label{fig:uep\_d}Simulated average PSNR with the EEP/UEP scheme.}

\end{figure}

In Fig.~\ref{fig:uep\_d}, it is shown that the UEP scheme has a significant performance gain in comparison to the EEP scheme when the encoder is given the same total transmission energy. It is worth noting that the gain is especially higher in the low SNR condition and then the gain gradually decreases with the growing of the SNR. Nevertheless, this trend is normal according to (\ref{eq:lagrange\_w\_n}) and (\ref{eq:lagrange\_constraint}), which imply that the larger the given $\gamma\_0$ is, the closer $\{\underbrace{1,\cdots,1}\_{m}\}$. That is

\begin{equation}

\{w\_n^{\ast}|n\in\{1,\cdots,m\}\}\stackrel{\text{ }\gamma\_0 \uparrow \text{ }}{\longrightarrow}\{\underbrace{1,\cdots,1}\_{m}\},

\label{eq:w\_n\_convergence}

\end{equation}

where $\{\underbrace{1,\cdots,1}\_{m}\}$ is just the power arrangement in the EEP scheme. As a result, the optimized quality performance is closer and closer to that of the EEP scheme since the given average transmission power is growing.

% Simulation EEP / UEP\_E

In addition to the first proposed UEP scheme in section \ref{ccc:problem-maxpsnr}, the other proposed UEP scheme in section \ref{ccc:problem-minpower}, which minimizes the total allocated transmission energy under the constraint on the target quality performance, is also simulated as the other application scenario. In this scenario, the encoder is required to achieve a given target of the least distortion $D\_0$ with the least total transmission energy. With a EEP scheme ($\{w\_n=1|n\in\{1,\cdots,m\}\}$), the energy allocation is determined by the aid of (\ref{eq:distortion\_est\_eep}),

\begin{equation}

\begin{aligned}

E\_0 = &N\_0\gamma\_0 \nonumber \\

=&N\_0 \cdot fr^{-1}(\frac{D\_0}{21845}),

\end{aligned}

\label{eq:energy\_est\_eep}

\end{equation}

where $fr^{-1}(\cdot)$ is the inverse function of $fr(\gamma)$. On the other hand, the proposed UEP scheme optimizes energy allocation according to (\ref{eq:lagrange\_E\_n}) - (\ref{eq:newton\_method\_function\_minE}), and the effective average channel SNR equals $\frac{\sum\_{n=1}^{m}{E\_n}}{m\cdot N\_0}$. Then, the simulation result of this scenario is depicted in Fig.~\ref{fig:uep\_e}.

\begin{figure}[!htb]

\centering

\includegraphics[width=\textwidth]{fig/psnr\_direct\_uep\_eep\_e.pdf}

\caption{\label{fig:uep\_e} Simulated average PSNR with the EEP/UEP scheme.}

\end{figure}

In Fig.~\ref{fig:uep\_e}, it is shown that the proposed UEP scheme can make better use of the transmission power than the EEP scheme since the UEP scheme saves significant energy at any given target quality. Also, the saved energy is decreasing with the growing of the required visual performance due to (\ref{eq:lagrange\_E\_n}), which implies that the weights gradually converge to the close values when $D\_0$ becomes larger. Besides, the closer the $\{E\_n|n\in\{1,\cdots,m\}\}$ is, the more similar the UEP scheme is with the EEP scheme. However, proposed UEP scheme still outperforms the EEP scheme throughout the different SNR conditions.

% conclusion

Finally, according to Fig.~\ref{fig:uep\_d} and Fig.~\ref{fig:uep\_e}, it is verified that the proposed UEP schemes indeed improve the contribution of transmission energy to the received video performance. It is worth noting that the two proposed UEP schemes are actually the different operation on the same quality-energy curve, and it can be proven by exchanging X and Y axis in Fig.~\ref{fig:upe\_e} and combing with Fig.~\ref{fig:uep\_d} to obtain Fig.~\ref{fig:same\_scheme}.

\begin{figure}[!htb]

\centering

\includegraphics[width=\textwidth]{fig/two\_uep\_same.pdf}

\caption{\label{fig:same\_scheme}The proposed UEP schemes in two scenario are actually operating on the same curve.}

\end{figure}

In Fig.~\ref{fig:same\_scheme}, two UEP curves in first and second scenario coincide with each other. That is, since the quality-SNR curve is one-to-one, two scenarios are actually the same concept from different viewpoints. As a consequence, the simulation in the following chapters only shows the operation in first scenario, where the encoder has to determine the weight of transmission energy allocation for each bit-plane when the average transmission energy is given and fixed.